Black Hole fusion made easy

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CQG 33, 155003 (2016)
arXiv:1603.00712

Iberian Relativity Meeting ERI 2016
Lisboa
\[ R_{\mu
u} - \frac{1}{2} g_{\mu
u} R = 8\pi G T_{\mu\nu} \]
What this talk is all about

\[ ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \]

13 Jan 1916
“mathematics, physics, and astronomy constitute one knowledge, ... which is only comprehended as a perfect whole”
In 1900 he put astronomical bounds on the curvature radius of space
64 light-years if hyperbolic
1600 light-years if elliptic
1914: volunteers for war
Belgium: weather station
France, Russia: artillery trajectories

March 1916: sent back home, ill with pemphigus—dies in May
“I made at once by good luck a search for a full solution. A not too difficult calculation gave the following result: ...”

\[ ds^2 = \left(1 - \frac{\gamma}{R}\right) dt^2 - \frac{dR^2}{1 - \frac{\gamma}{R}} - R^2(d\theta^2 + \sin^2 \theta d\phi^2) \]
A Einstein to K Schwarzschild
(early January 1915)

“I had not expected that one could formulate the exact solution of the problem in such a simple way. Next Thursday I shall present the work to the Academy”
Two more articles before he dies:
GR: interior solution for star
Quantum Theory
“Schwarzschild is a real loss. He would have been a gem, had he been as decent as he was clever”
Schwarzschild’s solution has been rediscovered many times over

J Droste May 1916    same coordinates
        (part of PhD thesis under Lorentz – first worked on 1913 *Entwurf* theory)

P Painlevé 1921, A Gullstrand 1922
    P-G coordinates (didn’t realize it was the same as Schw’s)

and others
Long, complex, painful path to correct interpretation
Black holes exist in Nature
Governed by conceptually simple and beautiful equations

\[ R_{\mu\nu} = 0 \]
Governed by conceptually simple and beautiful equations

\[ R_{\mu\nu} = 0 \]

but exceedingly hard to solve
Merger is most complicated of all

Involves non-linearity of Einstein’s equations at its most fiendish
Merger is most complicated of all

Involves non-linearity of Einstein’s equations at its most fiendish

or maybe not—not always
This is what you’d see (lensing)

Not a black hole, but its *shadow*
What is a black hole?

Spacetime region from which not even light can ever escape

Event Horizon
Collapsed Star

- outgoing light ray escapes
- outgoing light ray doesn't escape
- light ray separatrix
BLACK HOLE

What *can never be seen* from outside
(asymptotic infinite)
Null hypersurface

3-dimensional in 4-dimensional spacetime
Null hypersurface made of null geodesics (light rays)
caustic

where null geodesics enter to form part of event horizon
Event horizon is found by tracing a family of light rays in a given spacetime
Event horizon of binary black hole fusion
Event horizon of binary black hole fusion
Event horizon of binary black hole fusion

“pants” surface

Light rays that form the EH
line of **caustics**
where new lightrays enter to form part of the horizon
Event horizon of binary black hole fusion

head-on
(axisymmetric)
equal masses

Cover of Science, November 10, 1995

Binary Black Hole Grand Challenge Alliance (Matzner et al)
Event horizon of binary black hole fusion


Surely the fusion of horizons can only be captured with supercomputers
Surely the fusion of horizons can only be captured with supercomputers

or so it’d seem
∃ limiting (but realistic) instance where horizon fusion can be described exactly

It involves only elementary ideas and techniques
Equivalence Principle  (1907)

Schwarzschild solution & Null geodesics  (1916)

Notion of Event Horizon  (1950s/1960s)
Extreme-Mass-Ratio (EMR) merger

\[ m \ll M \]
EMR mergers in the Universe

\[ \frac{m}{M} \simeq \frac{1}{30} \] (or even less) may be detected with LIGO

\[ \frac{m}{M} \simeq 10^{-4} - 10^{-8} \] or less may be detected with LISA
Fusion of horizons involves scales $\sim m$

$m$ finite

$M \to \infty$
Very large black hole / Very close to the horizon

\[ M \rightarrow \infty \]
Very close to a Black Hole

Horizon well approximated by null plane in Minkowski space
This follows from the **Equivalence Principle**.

At short enough scales, geometry is equivalent to flat Minkowski space.

Curvature effects become small, but horizon remains.
Locally gravity is equivalent to acceleration

Locally black hole horizon is equivalent to acceleration horizon
Falling into very large bh = crossing a null plane in Minkowski space

in rest-frame of infalling object
Small Black Hole falling into a Large Black Hole

$M \to \infty$

$m$ fixed

Both are made of light rays

in rest frame of small black hole
Somehow, lightrays must merge to form a “pants-like” surface.

“oversized leg”

“thin leg”
EH is a family of lightrays in spacetime.

Curvature of small black hole is not zero: Schwarzschild solution with mass $m$.
To find the “pants” surface:
Trace a family of null geodesics in the Schwarzschild solution
that approach a null plane at infinity
All the equations you need to solve

\[ t_q(r) = \int \frac{r^3 \, dr}{(r-1)\sqrt{r(r^3-q^2(r-1))}} \]

\[ \phi_q(r) = \int \frac{q \, dr}{\sqrt{r(r^3-q^2(r-1))}} \]

with appropriate final conditions:

**null plane at infinity**

\( q = \text{impact parameter of light rays at infinity} \)
All the equations you need to solve

\[ t_q(r) = \int \frac{r^3 \, dr}{(r-1)\sqrt{r(r^3-q^2(r-1))}} \]

\[ \phi_q(r) = \int \frac{q \, dr}{\sqrt{r(r^3-q^2(r-1))}} \]

**elliptic integrals**

not very nice, but explicit
“Pants” surface

big black hole

small black hole

big black hole
caustic line
(crease set)
Sequence of constant-time slices

\[ t = -20r_0 \]

\[ t = -10r_0 \]

\[ t = -2r_0 \]

\[ t = -0.1r_0 \]

\[ t = 0 \] pinch-on

\[ t = r_0 \]

\[ t = 6r_0 \]

\[ t = 27r_0 \]

\[ r_0 = \text{small horizon radius} \]
Sequence of constant-time slices

$t = -2r_0$

$t = -0.1r_0$

$t = 0 : \text{pinch-on}$

$t = r_0$

$t = 6r_0$

$t = 27r_0$

$r_0 =$ small horizon radius
Sequence of constant-time slices

\[ t = -2r_0 \]

\[ t = -0.1r_0 \]

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\[ t = r_0 \]

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\( r_0 = \) small horizon radius

causitic cone
Pinch-on: Criticality
Pinch-on: Criticality

Opening angles of cones $\sim |t|^{1/2}$

same behavior in 5D: universal?
Pinch-on: Criticality

Throat growth \( \sim t \)

same behavior in 5D: universal?
Gravitational waves?

When $M \to \infty$ the radiation zone is pushed out to infinity

No gravitational waves in this region

GWs reappear if we introduce corrections for small $\frac{m}{M}$
Change Schwarzschild $\rightarrow$ Kerr

Fusion of **any Black Hole binary in the Universe**

to leading order in $\frac{m}{M} \ll 1$
Final remarks
Can we observe this?

Maybe not

Then, what is it good for?
Fusion of Black Hole Event Horizons is a signature phenomenon of General Relativity.

Equivalence Principle allows to capture and *understand* it easily in a (realistic) limit.
Exact result:

• Benchmark for detailed numerical studies

• First step in expansion in $\frac{m}{M} \ll 1$
  (matched asymptotic expansion)
Equivalence Principle magic:
Get 2 black holes out of a geometry with only 1

This could have been done (at least) 50 years ago!